Complete each of the problems below for both of the following affine semigroups:

$$S = \mathbb{N}\{2, 3\}$$
$$S = \mathbb{N}^2\{(4, 0), (3, 1), (1, 3), (0, 4)\}$$

(1) Compute the saturation $\overline{S}$ of S.

(2) Describe $\operatorname{Spec} \mathbb{C}[S]$ geometrically: What dimension is it? Can you embed it in an affine space?

(3) Describe $\operatorname{Spec} \mathbb{C}[\overline{S}]$. [hint: Look back at a previous worksheet and at textbook examples 1.1.6 and 1.1.22.]

(4) Define a map $\operatorname{Spec} \mathbb{C}[\overline{S}] \to \operatorname{Spec} \mathbb{C}[S]$.