(1) Suppose G is a (not necessarily abelian) group acting on a set X. Given another set Y, show that G also acts on the set of functions $\mathrm{Hom}(X, Y)$ by $(g \cdot f)(x) = f(g^{-1} \cdot x)$, where $g \in G$ and f and $g \cdot f$ are both functions $X \to Y$. Why is there an inverse in the formula?

(2) Suppose G acts on a finite dimensional vector space V so that for each fixed $g \in G$ multiplication by g is a linear map on V. Show that there is a group homomorphism $G \to GL(V)$, the group of invertible linear transformations $V \to V$.

(3) Let R be a commutative ring with elements $d, e \in R$ such that $d^2 = d$, $e^2 = e$, $de = 0$, and $d + e = 1_R$. Decompose R as $dR \oplus eR$ where dR is the ideal of elements which can be written in the form dr for some $r \in R$.

(4) Consider $\varphi\colon (\mathbb{C}^*)^2 \to GL_2(\mathbb{C})$ given by

$$(s,t) \mapsto \begin{bmatrix} \frac{s}{t} & t^3 - \frac{s}{t} \\ 0 & t^3 \end{bmatrix}.$$

Check that φ defines a group action on $\mathbb{C}^2$ and decompose $\mathbb{C}^2$ as a direct sum of two lines where $(\mathbb{C}^*)^2$ acts by characters.