

Here we will construct the Hirzebruch surfaces, whose fans have rays $(1, 0), (0, 1), (-1, a), (0, -1)$ for $a \in \mathbb{N}$.

- (1) Calculate the affine toric variety of each 2-dimensional cone.
- (2) Write ring maps for each of the inclusions of coordinate rings implied by the structure of the fan.
- (3) For $a = 0$ identify the points of the abstract toric variety X with the points $(c : d, f : g)$ of $\mathbb{P}^1 \times \mathbb{P}^1$.
- (4) Draw a fan for \mathbb{P}^1 .
- (5) For general a define a toric map $\varphi : X \rightarrow \mathbb{P}^1$.
- (6) Given a point $(c : d)$ of \mathbb{P}^1 , show that the preimage of $(c : d)$ under φ is also \mathbb{P}^1 , at least on closed points. [hint: Start with the case $a = 0$.]