When you learned about the affine toric variety of a rational polyhedral cone you saw that the toric variety X of the cone generated by $e_1, e_2, e_1 + e_3, e_2 + e_3$ in $\mathbb{R}^3$ is defined by the toric ideal $\langle xy - zw \rangle$. Describe the map $X \to \operatorname{Spec} \mathbb{C}[x, y, z, w]$ as ...

(1) a map of affine semigroups

(2) a map of semigroup rings extending to a map of group rings

(3) a map on closed points restricting to a group homomorphism on tori [possible hint: Describe X as the closure of the image of a map of tori.]

(4) a map on lattices restricting to the cones

Now suppose that σ is a strongly convex rational polyhedral cone and τ is a face of σ.

(5) Use whichever definition you like to construct a map from the affine toric variety of τ to the affine toric variety of σ.