Recall that a torus T has a group of characters M and a dual group $N = \mathrm{Hom}_{\mathbb{Z}}(M, \mathbb{Z})$. Elements of N are maps $\varphi\colon M \to \mathbb{Z}$.

Given $m \in M$ and $\varphi \in N$ we can calculate $\varphi(m) \in \mathbb{Z}$.

(1) If M has basis $m_1, \ldots, m_n$ and N has dual basis $\varphi_1, \ldots, \varphi_n$, calculate $\varphi(m)$ for $m = \sum a_i m_i$ and $\varphi = \sum b_i \varphi_i$.

Using dual bases this operation extends to the standard dot product on the vector spaces $M \otimes_{\mathbb{Z}} \mathbb{R}$ and $N \otimes_{\mathbb{Z}} \mathbb{R}$, which we will denote by $\langle -, - \rangle$.

(2) Verify that the dual of the cone spanned by $(0,1)$ and $(-1,2)$ is spanned by $(2,1)$ and $(-1,0)$ in the dual basis as claimed on the last worksheet.

(3) Show that $\sigma^\vee$ is a cone.

(4) Find an example of a cone whose dual is contained in it.