

A *character* of a torus  $T = \text{Spec}(\mathbb{C}[x_1^{\pm 1}, \dots, x_n^{\pm 1}])$  is a map of  $\mathbb{C}$ -algebras  $\mathbb{C}[t^{\pm 1}] \rightarrow \mathbb{C}[x_1^{\pm 1}, \dots, x_n^{\pm 1}]$  which induces a group homomorphism.

Start with  $n = 1$ .

(1) What are the maps of  $\mathbb{C}$ -algebras  $\mathbb{C}[t^{\pm 1}] \rightarrow \mathbb{C}[x^{\pm 1}]$ ?

(2) Show that your maps give group homomorphisms  $\mathbb{C}^* \rightarrow \mathbb{C}^*$ .

(3) Now show that if  $(\mathbb{C}^*)^n \rightarrow \mathbb{C}^*$  is a character of  $T$  then composing with the inclusion  $\mathbb{C}^* \rightarrow (\mathbb{C}^*)^n$  gives a character for each coordinate of  $(\mathbb{C}^*)^n$ .

(4) Construct a bijection between the characters of  $(\mathbb{C}^*)^n$  and the elements of  $\mathbb{Z}^n$ .