Let $\Phi\colon \mathbb{C}^* \to \mathbb{C}^2$ be given by $t \mapsto (t^2, t^3)$ and let X be the Zariski closure of the image of Φ (meaning the smallest subset of $\mathbb{C}^2$ which contains the image of Φ and is defined by polynomials).

(1) Draw a picture of the image of Φ.

(2) What additional point(s) are in the closure?

(3) Which ideal I defines X in $\mathbb{C}^2$? (Show that the functions in your I vanish on X but don't worry about the converse.)

(4) Write $V(I) \subset \operatorname{Spec}(\mathbb{C}[x,y])$ as Spec of a ring R.

(5) Describe R as an affine semigroup ring.

(6) The map Φ restricts to a homomorphism of tori $\mathbb{C}^* \to (\mathbb{C}^*)^2$, and $(\mathbb{C}^*)^2$ acts on $\mathbb{C}^2$ by $(s,t)\cdot(x,y) = (sx, ty)$. What is the induced action of $\mathbb{C}^*$ on $\mathbb{C}^2$?

(7) Does the action of $\mathbb{C}^*$ restrict to X?