

An affine toric variety $X = \text{Spec } R$ has the action of a torus $T = \text{Spec}(\mathbb{C}[x_1^{\pm 1}, \dots, x_n^{\pm 1}])$ and contains T as an open subset. The group action comes from an algebraic map $T \times X \rightarrow X$ which restricts to multiplication $T \times T \rightarrow T$. (Here \times means $\times_{\mathbb{C}}$, and R is a domain.)

(1) Write $T \times X$ and $T \times T$ as spectra of rings.

(2) Describe the multiplication map $T \times T \rightarrow T$ as a map of rings.

(3) What does it mean about R that X contains T as an open subset?

(4) How would you check compatibility of the action with multiplication?